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MINIMUM DOMINATING SEIDEL ENERGY OF A GRAPH

M.R. RAJESH KANNA*, R JAGADEESH, B.K. KEMPEGOWDA

Abstract—In This paper, we introduce the concept of minimum dominating seidel energy of a graph $SE_D(G)$ and computed minimum dominating seidel energy of a star graph, complete graph crown graph and cocktail party graphs. Upper and lower bounds for $SE_D(G)$are established.

Mathematics Subject Classification*:* Primary 05C50, 05C69.

Keywords and Phrases*:*Minimum dominating Seidel set, Minimum dominating Seidel matrix, Minimum dominating Seidel eigenvalues, Minimum dominating Seidel energy of a graph.

1 INTRODUCTION

The concept of energy of a graph was introduced by I. Gutman[7] in the year 1978. Let G be a graph with n vertices and m edges and let A =(a_{ij}) be the adjacency matrix of the graph. The eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots \lambda_n$ of A, assumed in non increasing order, are the eigenvalues of a graph G. As A is real symmetric, the eigenvalues of G are real with sum equal to zero. The energy E(G) of G is defined to be sum of the absolute values of eigenvalues of G . i.e., $E(G) = \sum_{i=1}^{n} |\lambda_i|$.

For details on the mathematical aspects of the theory of graph energy seethe reviews[8], paperr[4, 5, 9] and references cited there in. The basic properties including various upper and lower bounds for energy of a graph have been established in [11, 13] and it has found remarkable chemical applications in the molecular orbital theory of conjugated molecules [6, 10]. Further studies on covering energy and dominating energy can be found in [3, 15].

1.1 SEIDEL ENERGY

Let G be a simple graph of order n with verrtex set V = {v_1, v_2, v_3, ……v_n} and ege set E. The Seidel matrix of G is the n x n matrix defined by S(G):= (s_{ij}), where

$$\text{where } s_{ij} = \begin{cases} -1 & \text{if } v_i v_j \in E \\ 1 & \text{if } v_i v_j \notin E \\ 0 & \text{if } v_i = v_j \end{cases}$$

The characteristic polynomial of S(G) is denoted by $f_n(G, \lambda) = \det(\lambda I - S(G))$. The seidel eigenvalues of a graph G are the eigenvalues of S(G). since S(G) is real and symmetric, its eigenvalues are real numbers. The Seidel energy [16] of G is defined as $SE(G) = \sum_{i=1}^{n} |\lambda_i|$.

1.2 MINIMUM DOMINATING SEIDEL ENERGY

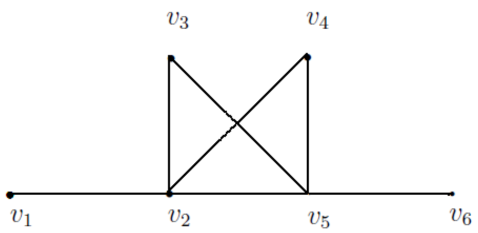
Let G be a simple graph of order n with verrtex set V = {v_1, v_2, v_3, ……v_n} and edge set E. A subset D of V is called a dominating set of G if every verrtex of V-D is adjacent to some vertex in D. Any dominating set with minimum cardinality is called minimum dominating set. The minimum dominating seidel matrix of G is the n x n matrix defined by $S_D(G)$:= (s_{ij}),

$$\text{where } s_{ij} = \begin{cases} -1 & \text{if } v_i v_j \in E \\ 1 & \text{if } v_i v_j \notin E \\ 1 & \text{if } i = j \text{ and } v_i \in D \\ 0 & \text{if } i = j \text{ and } v_i \notin D \end{cases}$$

The characteristic polynomial of $S_D(G)$ is denoted by $f_n(G, \lambda) = \det(\lambda I - S(G))$. The minimum dominating seidel eigenvalues of a graph G are the eigenvalues of $S_D(G)$. since $S_D(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \ldots \ldots . \lambda_n$.
The minimum dominating Seidel energy of G is defined as . $SE_D(G) = \sum_{i=1}^{n} |\lambda_i|$

Note that the trace of $SE_D(G)$ = Dominating Number = k.

EXAMPLE1:
The possible minimum dominating sets for the following graph G are i) D_1 = {v_1, v_5} ii) D_2 = {v_2, v_5} iii) D_3 = {v_2, v_6}



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i) $$S_D(K_n)=\begin{pmatrix} 1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & 0 & -1 & \cdots & -1 & -1 \\ -1 & -1 & 0 & \cdots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \cdots & -1 & 0 \end{pmatrix}_{n\times n}$$

ii) $$S_{D1}(G)=\begin{pmatrix} 1 & -1 & 1 & 1 & 1 & 1 \\ -1 & 0 & -1 & -1 & -1 & 1 \\ 1 & -1 & 0 & 1 & -1 & 1 \\ 1 & -1 & 1 & 0 & -1 & 1 \\ 1 & -1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 0 \end{pmatrix}$$

Characteristic equation is

$$\lambda^6 - 2\lambda^5 - 14\lambda^4 + 12\lambda^3 + 57\lambda^2 + 14\lambda - 20 = 0.$$

The minimum dominating Seidel eigenvalues are

$\lambda_1 \approx 0.47432562456252$, $\lambda_2 \approx -1.0$,

$\lambda_3 \approx -1.614540819326829$,

$\lambda_4 \approx 2.83396262531981$,

$\lambda_5 \approx -2.452013091804278$, $\lambda_6 \approx 3.7582661248777$.

The minimum dominating Seidel energy $SE_{D1}(G) \approx 12.13310782226221$.

ii) $$S_{D2}(G)=\begin{pmatrix} 0 & -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 0 & 1 & -1 & 1 \\ 1 & -1 & 1 & 0 & -1 & 1 \\ 1 & -1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & 0 \end{pmatrix}$$

Characteristic equation is

$$\lambda^6 - 2\lambda^5 - 14\lambda^4 + 12\lambda^3 + 61\lambda^2 + 14\lambda - 24 = 0.$$

The minimum dominating Seidel eigenvalues are

$\lambda_1 \approx 0.51513804712807$, $\lambda_2 \approx -0.99999999999999$,

$\lambda_3 \approx -2.0$, $\lambda_4 \approx 3.0$, $\lambda_5 \approx -2.141336115655365$,

$\lambda_6 \approx 3.626198068527294$.

The minimum dominating Seidel energy

$SE_{D2}(G) \approx 12.28267223131073$.

Therefore, Minimum Dominating Seidel energy depends on the dominating set.

2. MINIMUM DOMINATING SEIDEL ENERGY OF SOME STANDARD GRAPHS

Theorem 2.1. For n≥2, minimum dominating seidel energy of complete graph K_n is $(n-2)+\sqrt{n^2+2n-3}$.

Proof. K_n is a complete graph with vertex set V = {v_1, v_2, v_3, ……v_n}. The minimum dominating set is D = {v_1}.Then

Characteristic equation is

$$(-1)^n(\lambda-1)^{n-2}[\lambda^2+(n-3)\lambda-(2n-3)] = 0.$$

The minimum dominating Seidel eigenvalues are

$\lambda = 1$ [(n-2) times], $\lambda = \dfrac{(n-3)\pm\sqrt{n^2+2n-3}}{2}$ [one time each].

Minimum dominating Seidel energy, $SE_D(K_n)$

$$= |1|(n-2) + \left|\frac{(n-3)+\sqrt{n^2+2n-3}}{2}\right| + \left|\frac{(n-3)-\sqrt{n^2+2n-3}}{2}\right|$$

$$=(n-2)+\sqrt{n^2+2n-3}.$$

Definition 2.1. *The Cocktail party graph is denoted by $K_{n\times 2}$, is a graph having the vertex set $V = \bigcup_{i=1}^{n}\{u_i, v_i\}$ and the edge set $E = \{u_iu_j, v_iv_j : i \neq j\}\bigcup\{u_iv_j, v_iu_j : 1 \leq i < j \leq n\}$.*

Theorem 2.2. Then minimum dominating Seidel energy of cocktail party graph $K_{n\times 2}$, for n≥ 2 is (4n 7) + √$4n^2$ + 4n – 7.

Proof. Let $K_{n\times 2}$ be the cocktail party graph with vertex set V = U {u_i, v_i } . The minimum dominating set is D = { u_1, v_1 }. Then

$$S_D(K_{n\times 2}) = \left(\begin{array}{c|ccccc|ccccc} & u_1 & u_2 & u_3 & \dots & u_n & v_1 & v_2 & v_3 & \dots & v_n \\ \hline u_1 & 1 & -1 & -1 & \dots & -1 & 1 & -1 & -1 & \dots & -1 \\ u_2 & -1 & 0 & -1 & \dots & -1 & -1 & 1 & -1 & \dots & -1 \\ u_3 & -1 & -1 & 0 & \dots & -1 & -1 & -1 & 1 & \dots & -1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ u_n & -1 & -1 & -1 & \dots & 0 & -1 & -1 & -1 & \dots & 1 \\ \hline v_1 & 1 & -1 & -1 & \dots & -1 & 1 & -1 & -1 & \dots & -1 \\ v_2 & -1 & 1 & -1 & \dots & -1 & -1 & 0 & -1 & \dots & -1 \\ v_3 & -1 & -1 & 1 & \dots & -1 & -1 & -1 & 0 & \dots & -1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ v_n & -1 & -1 & -1 & \dots & 1 & -1 & -1 & -1 & \dots & 0 \end{array}\right)$$

$$\lambda(\lambda+1)^{n-1}(\lambda-3)^{(n-2)}[(\lambda^2+(2n-7)\lambda-2(4n-7)] = 0$$

Characteristic equation is

Minimum dominating Seidel eigenvalues are λ = 0 [one time],

$\lambda = 3$ [$(n-2)$ times], $\lambda = \dfrac{(2n-7)\pm\sqrt{4n^2+4n-7}}{2}$ [one time each]

λ = - 1 [(n – 1) times],

$$= 0 + |-1|(n-1) + |3|(n-2) + \left|\frac{(2n-7)+\sqrt{4n^2+4n-7}}{2}\right| + \left|\frac{(2n-7)-\sqrt{4n^2+4n-7}}{2}\right|$$

$$=1(n-1)+3(n-2)+\sqrt{4n^2+4n-7}$$

$$=(4n-7)+\sqrt{4n^2+4n-7}.$$

Minimum dominating Seidel energy, $SE_D(K_{n\times 2})$

Theorem 2.3. for n ≥ 2 , the minimum dominating Seidel energy of Star graph $K_{1,n-1}$ is equal to (n -2) + √n^2 – 2n +5.

$$S_D(K_{1,n-1}) = \begin{pmatrix} 1 & -1 & -1 & \dots & -1 \\ -1 & 0 & 1 & \dots & 1 \\ -1 & 1 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 1 & 1 & \dots & 0 \end{pmatrix}_{n\times n}$$

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Proof. Consider the star graph $K_{1,n-1}$ with vertex set V = { v_o, v_1 ,v_2 ...., v_{n-1}} . Minimum dominating set is D = {v_o}. Then

Characteristic equation is

$$(-1)^n(\lambda+1)^{n-2}[\lambda^2-(n-1)\lambda-1]=0.$$

The minimum dominating Seidel eigenvalues are

λ = -1 [(n-2) times], $\lambda = \frac{(n-1)\pm\sqrt{n^2-2n+5}}{2}$ [one time each].

The minimum dominating Seidel energy is,

$$SE_D(K_{1,n-1}) = |-1|(n-2)+\left|\frac{(n-1)+\sqrt{n^2-2n+5}}{2}\right|+\left|\frac{(n-1)-\sqrt{n^2-2n+5}}{2}\right|$$

$$=(n-2)+\sqrt{n^2-2n+5}$$

Definition 2.2. The Crown graph S^o_n for an integer n ≥ 2 is the graph with vertex set {u_1 , u_2, …, u_n , v_1 , v_2, …, u_n } and edge set {u_i v_j : 1 ≤ i , j ≤ n , i ≠ j }. ∴ S^o_n coinciedes with the Complete bi[artite graph $K_{n,n}$ with horizontal edges removed.

Theorem 2. 4. for n ≥ 2 , the minimum dominating Seidel Energy of the Crown graph S^o_n is equal to (4n - 5) + √4n^2 - 4n + 9 .

Proof. For the Crown graph S^o_n with vertex set V = { u_1 , u_2 , …, u_n, v_1, v_2 , …, v_n}, Minimum dominating set S = {u_1 , v_1} Then,

$$S_D(S_n^0)=\left(\begin{array}{c|ccccc|ccccc} & u_1 & u_2 & u_3 & \ldots & u_n & v_1 & v_2 & v_3 & \ldots & v_n \\ \hline u_1 & 1 & 1 & 1 & \ldots & 1 & 1 & -1 & -1 & \ldots & -1 \\ u_2 & 1 & 0 & 1 & \ldots & 1 & -1 & 1 & -1 & \ldots & -1 \\ u_3 & 1 & 1 & 0 & \ldots & 1 & -1 & -1 & 1 & \ldots & -1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ u_n & 1 & 1 & 1 & \ldots & 0 & -1 & -1 & -1 & \ldots & 1 \\ \hline v_1 & 1 & -1 & -1 & \ldots & -1 & 1 & 1 & 1 & \ldots & 1 \\ v_2 & -1 & 1 & -1 & \ldots & -1 & 1 & 0 & 1 & \ldots & 1 \\ v_3 & -1 & -1 & 1 & \ldots & -1 & 1 & 1 & 0 & \ldots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ v_n & -1 & -1 & -1 & \ldots & 1 & 1 & 1 & 1 & \ldots & 0 \end{array}\right)_{(2n\times 2n)}$$

Characteristic equation is

$$(\lambda-2)(\lambda-1)^{n-1}(\lambda+3)^{n-2}[\lambda^2-(2n-5)\lambda-4(n-1)]=0$$

Minimum dominating Seidel eigenvalues are

$$\lambda=2[\text{one time}],\quad \lambda=1[(n-1)\text{times}],\ \lambda=-3[(n-2)\text{times}],$$
$$\lambda=\frac{(2n-5)\pm\sqrt{4n^2-4n+9}}{2}\ [\text{one time each}].$$

Minimum dominating Seidel energy, $SE_D(S^o_n)$

$$=2(1)+1(n-1)+|-3|(n-2)+\left|\frac{(2n-5)+\sqrt{4n^2-4n+9}}{2}\right|+\left|\frac{(2n-5)-\sqrt{4n^2-4n+9}}{2}\right|$$

$$=(4n-5)+\sqrt{4n^2-4n+9}.$$

3. PROPERTIES OF MINIMUM DOMINATING SEIDEL EIGENVALUES

Theorem 3. Let G be a simple graph with vertex set V = {v_1, v_2, **…,** v_n **},** edge set E and D = {u_1, u_2, ... u_k} be a minimum dominating set. If $\lambda_1, \lambda_2, \ldots\lambda_n$ are the eigenvalues of minimum dominating Seidel matrix S_D (G) then,

$$(i)\ \sum_{i=1}^{n}\lambda_i = |D|.$$

$$(ii)\ \sum_{i=1}^{n}\lambda_i^2 = |D|+n^2-n.$$

Proof . i) We know that the sum of the eigenvalues of S_D (G) is the trace of S_D (G)

$$\therefore \sum_{i=1}^{n}\lambda_i=\sum_{i=1}^{n}a_{ii}=|D|=k.$$

(ii) Similarly the sum of squares of the eigenvalues of S_D (G) is the trace of [$S_D(G)^2$]

$$\therefore \sum_{i=1}^{n}\lambda_i^2=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}a_{ji}$$
$$=\sum_{i=1}^{n}(a_{ii})^2+\sum_{i\neq j}a_{ij}a_{ji}$$
$$=\sum_{i=1}^{n}(a_{ii})^2+2\sum_{i<j}(a_{ij})^2$$
$$=|\,D\,|+2\Big[m(-1)^2+\Big(\frac{n^2-n}{2}-m\Big)(1)^2\Big]$$
$$=|\,D\,|+n^2-n.$$

4. BOUNDS FOR MINIMUM DOMINATING SEIDEL ENERGY

Similar to McClelland's [13] bounds for energy of a graph, bounds for SE_D (G) are given in the following theorem.

Theorem 4.1 Let G be a simple graph with n vertices and m edges . If D is the minimum dominating set and P = |detS_D (G)| then
√(n^2 - n + k) + n (n – 1) $P^{2/n}$ ≤ SE_D (G) ≤ √n(n^2 – n + k) where k is a domination number.

Proof.
Since arithmetic mean is not smaller than geometric mean we have

Cauchy Schwarz inequality is $\left(\sum_{i=1}^{n}a_ib_i\right)^2\le\left(\sum_{i=1}^{n}a_i^2\right)\left(\sum_{i=1}^{n}b_i^2\right)$

If $a_i=1, b_i=|\,\lambda_i\,|$ then $\left(\sum_{i=1}^{n}|\,\lambda_i\,|\right)^2\le\left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}\lambda_i^2\right)$

$$[SE_D(G)]^2\le n(n^2-n+k)\qquad[\text{Theorem 3.1}]$$
$$\Longrightarrow SE_D(G)\le\sqrt{n(n^2-n+k)}$$

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\lambda_i||\lambda_j| \geq \left[\prod_{i\neq j}|\lambda_i||\lambda_j|\right]^{\frac{1}{n(n-1)}}$$
$$= \left[\prod_{i=1}^{n}|\lambda_i|^{2(n-1)}\right]^{\frac{1}{n(n-1)}}$$
$$= \left[\prod_{i=1}^{n}|\lambda_i|\right]^{\frac{2}{n}}$$
$$= \left|\prod_{i=1}^{n}\lambda_i\right|^{\frac{2}{n}}$$
$$= |detS_D(G)|^{\frac{2}{n}} = P^{\frac{2}{n}}$$

$$\therefore \sum_{i\neq j}|\lambda_i||\lambda_j| \geq n(n-1)P^{\frac{2}{n}} \qquad (4.1)$$

Now consider,

$$[SE_D(G)]^2 = \left(\sum_{i=1}^{n}|\lambda_i|\right)^2$$
$$= \sum_{i=1}^{n}|\lambda_i|^2 + \sum_{i\neq j}|\lambda_i||\lambda_j|$$

$$\therefore \quad [SE_D(G)]^2 \geq (k+n^2-n) + n(n-1)P^{\frac{2}{n}} \qquad \text{[From (4.1)]}$$
$$i.e., \; SE_D(G) \geq \sqrt{(k+n^2-n)+n(n-1)P^{\frac{2}{n}}}$$

Therorem 4.2. If $\lambda_1(G)$ is the largest minimum dominating Seidel eigenvalue of S_D (G), then

$$\lambda_1(G) \geq \frac{n^2-n+k}{n}$$ where k is the domination number.

Proof . Let X be any nonzero vector. Then by [1] , We have

$$\lambda_1(A) = \max_{X\neq 0}\left\{\frac{X'AX}{X'X}\right\}$$

$$\therefore \lambda_1(A) \geq \frac{J'AJ}{J'J} = \frac{n^2-n+k}{n} \text{ where } J \text{ is a unit matrix } [1,1,1,...,1]'.$$

Similar to Koolen and Moulton's [12] upper bound for energy of a graph, upper bound for SE_D (G) is given in the following theorem.

Theorem 4.3. If G is a graph with n vertices and m edges and $(n^2 - n + k) \geq n$ then $SE_D(G) \leq n^2 - n + k + \sqrt{(n-1)[(n^2-n+k)-(n^2-n+k)^2]}$ where k is a domination number.

Proof:

Cauchy-Schwartz inequality is $\left[\sum_{i=2}^{n}a_ib_i\right]^2 \leq \left(\sum_{i=2}^{n}a_i^2\right)\left(\sum_{i=2}^{n}b_i^2\right)$

Put $a_i = 1, b_i = |\lambda_i|$ then $\left(\sum_{i=2}^{n}|\lambda_i|\right)^2 = \sum_{i=2}^{n}1\sum_{i=2}^{n}\lambda_i^2$

$$\Rightarrow [SE_D(G)-\lambda_1]^2 \leq (n-1)(n^2-n+k-\lambda_1^2)$$
$$\Rightarrow SE_D(G) \leq \lambda_1 + \sqrt{(n-1)(n^2-n+k-\lambda_1^2)}$$

Let $f(x) = x + \sqrt{(n-1)(n^2-n+k-x^2)}$

For decreasing function $f'(x) \leq 0 \Rightarrow 1 - \frac{x(n-1)}{\sqrt{(n-1)(n^2-n+k-x^2)}} \leq 0$

$$\Rightarrow x \geq \sqrt{\frac{n^2-n+k}{n}}$$

Recently Milovanovic [14] et al. gave a sharper lower bounds for energy of a graph. In this paper similar bounds for minimum dominating Seidel energy of a graph are established.

Since $(n^2-n+k) \geq n$, we have $\sqrt{\frac{n^2-n+k}{n}} \leq \frac{n^2-n+k}{n} \leq \lambda_1$ [From theorem 4.2]

$\therefore f(\lambda_1) \leq f\left(\frac{n^2-n+k}{n}\right)$

$i.e., SE_D(G) \leq f(\lambda_1) \leq f\left(\frac{n^2-n+k}{n}\right)$

$i.e., SE_D(G) \leq f\left(\frac{n^2-n+k}{n}\right)$

$i.e., SE_D(G) \leq \frac{n^2-n+k}{n} + \sqrt{(n-1)\left[n^2-n+k-\left(\frac{n^2-n+k}{n}\right)^2\right]}.$

Theorem 4.4. Let G be a graph with n vertices and m edges. Let $|\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_n|$ be a non – increasing order of minimum dominating seidel eiqenvalues of S_D (G) and D is minimum dominating set then $SE_D(G) \geq \sqrt{n(n^2-n+|D|) - \alpha(n)(|\lambda_1|-|\lambda_n|)^2}$ where α (n) =n [n/2] (1 –1/n[n/2]) and [x]denotes the integral part of a real number.

Proof. Let $a, a_1, a_2, \ldots, a_n$, A and b, $b_1, b_2, \ldots, b_n$, B be real numbers such that $a \leq a_i \leq A$ and $b \leq b_i \leq B$ for all i = 1,2,. . . n then the following inequality is valid.

$\left|n\sum_{i=1}^{n} a_ib_i - \sum_{i=1}^{n}a_i \sum_{i=1}^{n}b_i\right| \leq \alpha(n)(A-a)(B-b)$ where α(n)= n (n/2) (1 -1/n [n/2]) and equality holds if and only if $a_1 = a_2 = \ldots = a_n$ and $b_1 = b_2 = \ldots = b_n$.

If $a_i = |\lambda_i|$, $b_i = |\lambda_i|$, $a = b = |\lambda_n|$ and $A = B = |\lambda_1|$; then

$$\left|n\sum_{i=1}^{n}|\lambda_i|^2 - \left(\sum_{i=1}^{n}|\lambda_i|\right)^2\right| \leq \alpha(n)(|\lambda_n|^2$$

But $\sum_{i=1}^{n}|\lambda_i|^2 = n^2 - n + |D|$ and $SE_D(G) \leq \sqrt{n(n^2-n+|D|)}$ then the above inequality becomes

$n(n^2-n+|D|) - (SE_D(G))^2 \leq \alpha(n)(|\lambda_1| - |\lambda_n|)^2$

i,e., $SE_D(G) \geq \sqrt{n(n^2-n+|D|) - \alpha(n)(|\lambda_1|-|\lambda_n|)^2}$

Theorem 4.5. Let G be a graph with n vertices and m edges. Let $||\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_n| > 0$ be a non-increasing order of eigenvalues of S_D (G) then

$$SE_D(G) \geq \frac{n^2-n+|D|+n|\lambda_1||\lambda_n|}{(|\lambda_1|+|\lambda_n|)}$$

Proof. : Let $a_i \neq 0$, b_i, r and R be real numbers satisfying $ra_i \leq b_i \leq Ra_i$, then the following inequality holds. [Theorem 2, [14]]

$$\sum_{i=1}^{n} b_i^2 + rR\sum_{i=1}^{n}a_i^2 \leq (r+R)\sum_{i=1}^{n} a_ib_i$$

Put $b_i = |\lambda_i|$, $a_i = 1$, $r = |\lambda_n|$ and $R = |\lambda_1|$ then

$$\sum_{i=1}^{n}|\lambda_i|^2 + |\lambda_1||\lambda_n|\sum_{i=1}^{n}1 \leq (|\lambda_1|+|\lambda_n|)\sum_{i=1}^{n}|\lambda_i|$$
$$i.e., \; n^2-n+|D|+|\lambda_1||\lambda_n|n \leq (|\lambda_1|+|\lambda_n|)SE_D(G)$$
$$\therefore SE_D(G) \geq \frac{n^2-n+|D|+n|\lambda_1||\lambda_n|}{(|\lambda_1|+|\lambda_n|)}$$

Bapat and S.pati [2] proved that if the graph energy is a rational number then it is an even integer. Similar result for minimum dominating Seidel energy is given in the following theorem.

Theorem 4.6. Let G be a graph with a minimum dominating set D. IF the minimum dominating Seidel energy SE_D (G) is a rational number, then $SE_D(G) \equiv |D| \pmod{2}$.

Proof. The proof is similar to the theorem 5.4 of [15]

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